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ABSTRACT

A pilot study was conducted to examine the information with respect to the modes, referents, and relationships that a given teacher acquires through the examination of her students' written responses to open-ended tasks and how the teacher uses this information. The participating teacher taught sixth grade in a public elementary school in a suburban neighborhood. The teacher had been trained in the rating of open-ended tasks according to pre-established criteria as part of a larger project. Tasks requiring computing areas and averaging were administered to students. The teacher was asked to predict how students would perform on each task and to suggest the approaches they would use. The teacher was later interviewed as she examined students' responses to the tasks to complete a Task Knowledge Inventory. The analysis of the teacher's responses suggests that she did have an adequate understanding of the concepts that were assessed in each task. She was aware that the students would use different approaches. The students' responses did offer the teacher information she did not previously possess. She was not able to predict accurately the methods the students would use to solve these problems, and she gained information about the depth of their knowledge and the referents they used in problem solving. The bulk of the teacher's attention during the students' acquisition process was on the individual, but her discussion of potential uses was focused on the larger group for instructional decision making. Results suggest that the teacher actually acquired more information about the students through the examination process than she used, especially with regard to students' communication processes. (Contains 6 tables, 8 figures, and 30 references.) (SLD)

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ED 409 379

Open-Ended Mathematics Tasks:

How did a Middle School Teacher Interpret and Use Information Acquired through the Examination of Student Responses?

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Introduction

Classroom assessment has become a topic of increasing interest among mathematics researchers (Webb, 1992; Romberg, Zarinnia & Collis, 1990) and practitioners (Wiggins, 1989; Thompson & Briers, 1989). Both groups have recently devoted much discussion to various techniques of assessment. For example, the Pennsylvania Council of Teachers of Mathematics (1995) dedicated their forty-fourth annual meeting to the discussion and exchange of ideas for classroom assessment. Workshops, mini courses and presentations provided information on a variety of techniques, such as observation, portfolios, interviews, open-ended tasks and extended problem situations. Books (e.g., Kulm, 1994; Lesh & Lamon, 1992a), professional articles (e.g., Wiggins, 1988; 1989; Thompson & Briers, 1989) and World Wide Web Sites (e.g., http://www.exit109.com/~learn/mathases.html; http://www.lhs.berkeley.edu/EQL102.html) have also focused attention on assessment issues.

One assessment instrument that has received much of this attention is the open-ended task. Open-ended refers to a task structure which allows students to determine their own approach when solving problems. These tasks typically include a request for a display of either the student's approach or reasoning process (van den Heuvel-Panhuizen, 1995; van Reeuwijk, 1995; Resnick, 1988). Open-ended tasks which vary in time requirements may be administered to students in written or oral form and may be completed by students individually or in groups. Problems that emerge outside the school setting are likely to be open-ended (Resnick, 1988). Since the requirements of open-ended tasks may resemble realistic problems that are external to the school setting, activities that appear in open-ended form are considered to be appropriate to the preparation of students as contributing members of society (National Research Council, 1989; NCTM, 1989; 1991; 1995).

Past research regarding the use of written open-ended tasks to assess students' knowledge suggests that the examination of students' responses for the communication methods that are utilized, the mathematical systems that are employed and the connections that are displayed provide evidence that suggests how the student is making-sense of a problem. *Modes*, which are the manner in which students select to communicate their knowledge, are believed to be a reflection of the student's internal representations (Lesh & Lamon, 1992b; Streefland, 1992; Hiebert & Carpenter, 1992). *Referent systems*, or the objects that may be external to the designated mathematical system that students employ in solving a problem, suggest the underlying reasoning that supports a solution (Resnick, 1988; Resnick, Nesher, Leonard, Magone, Omanson & Peled, 1989). The *relationships* students make among concepts provide evidence of some, but not all, of the mathematical connections that a given student has established (Burton, 1986). By combining the information acquired through

the examination of modes, referents and relationships, the assessor is likely to obtain an accurate depiction of the student's sense-making process. Most assessments take place as part of instruction and the interpreter is the classroom teacher (Stiggins, 1993). Teachers commonly use the information that they acquire to assist in instructional decision making (Clark & Peterson, 1986; Barr, 1988; Shavelson & Stern, 1981) and to provide feedback to parents, administrators, students and other teachers (Webb, 1992; Shavelson & Stern, 1992).

The bulk of research that concerns the interpretation and use of assessment information acquired through open-ended tasks has focused on the efforts of external assessors rather than classroom teachers. Classroom teachers have typically received less training in assessment pedagogy than have external assessors (Stiggins, 1990; 1991). Teachers also have personal experiences with students that influence the interpretation process (Webb, 1992). Because of these differences, one cannot assume that teachers will acquire the same information as do external assessors when interpreting a given set of student responses. If open-ended tasks are to be introduced to the classroom, research is needed to determine what information teachers acquire through the use of open-ended tasks and how they use this information to serve classroom purposes.

The study reported here is a pilot to a larger study. The purpose of this pilot study is to examine the information with respect to modes, referents and relationships that a given teacher acquires through the examination of her students' written responses to open-ended tasks. The research questions that guide this investigation are:

1) What information with respect to modes, referents and relationships did the respective teacher acquire through the use of written open-ended tasks?

2) How did this teacher report she would use the information acquired concerning modes, referents and relationships from written open-ended tasks for decision making and feedback purposes?

Methods

Subject

The participating teacher taught sixth grade in a public elementary school located in a suburban neighborhood. The participating teacher had the unique experience of being trained in the rating of open-ended tasks according to pre-established criteria as part of the Quantitative Understanding: Amplifying Student

Achievement and Reasoning (QUASAR) project's[1] rating sessions. Additionally, this teacher holds a master's degree in elementary education with a specialization in mathematics.

At the time of this study, the teacher had 8 years of teaching experience and she was responsible for providing instruction in mathematics, reading, social studies, spelling, English, and science. Although the composition of individuals who attended each class varied by subject, the majority of students had this teacher for more than one subject.

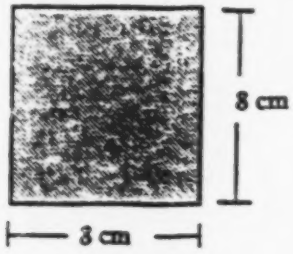
Instruments

Tasks

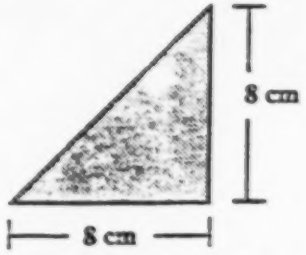
The area task shown in figure 1 was administered to students in April of 1995. A month later, the average task shown in figure 2 was administered. These tasks were selected from a pool of QUASAR released tasks to be consistent with the content that had been taught within a week prior to task administration.

Figure 1. Area Task

Teressa was working on an art project. Shown below are a square and a triangle she cut out.

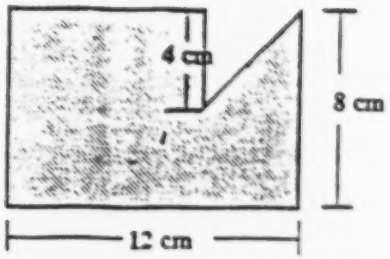


Area is 64 square centimeters



Area is 32 square centimeters

She glued part of the triangle on top of part of the square. The new shape is shown below.



What is the area of the new shape? Explain how you found your answer. You may use the drawings in your explanation.

[1] QUASAR is a project that seeks to examine the impact of reform-oriented instruction on middle school students' development of thinking and reasoning skills (Silver, 1993; Lane & Silver, 1994). Middle school teachers are trained through "rating sessions" to score the collected students' responses to open-ended tasks according to pre-established criterea.

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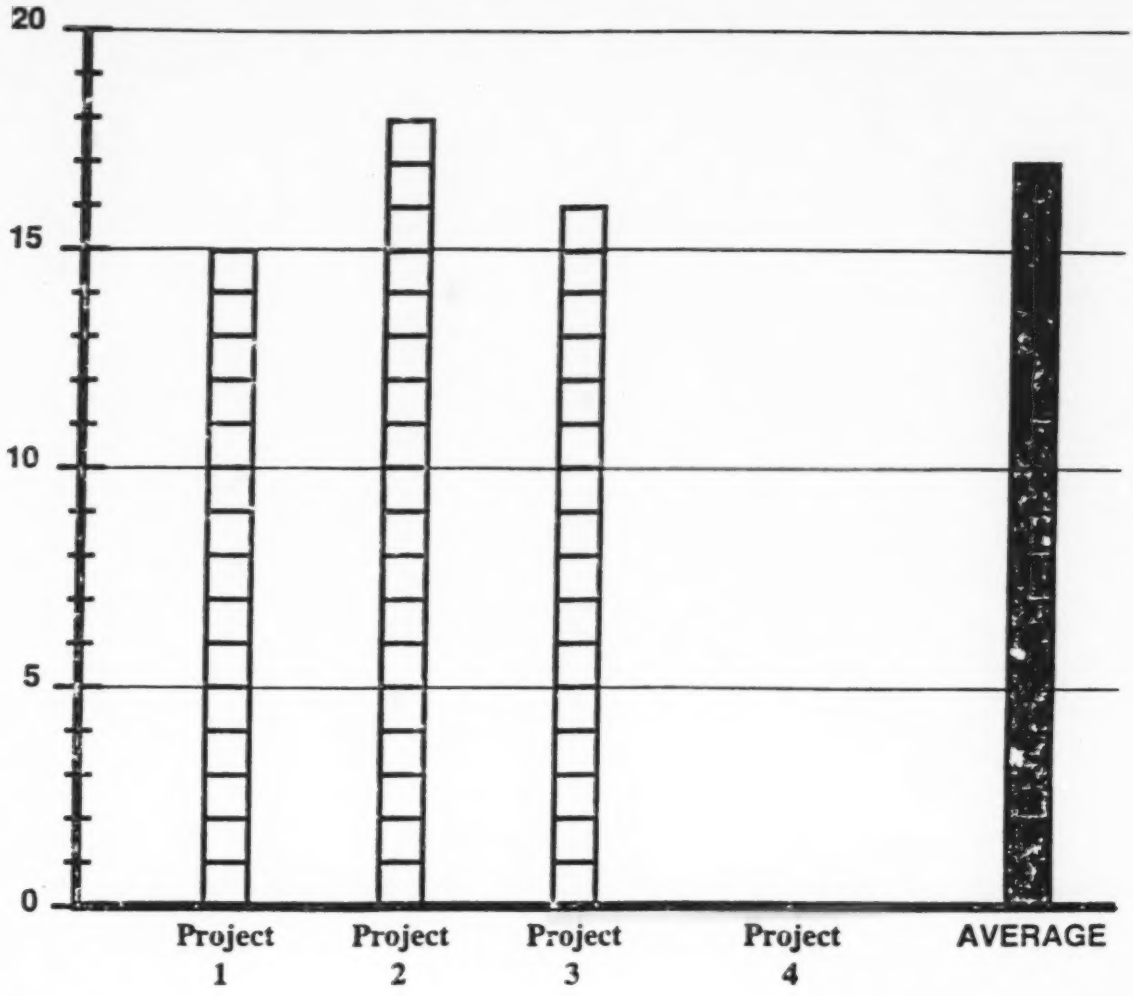
Figure 2. Average Task

Anita has four 20-point projects for science class. Anita's scores on the first 3 projects are shown below.



A. What score must Anita get on Project 4 so that her average for the four projects is 17?

Answer: _____ You may draw your answer on the graph.

B. Explain how you found your answer.

Background Survey

The background survey was designed to collect descriptive information concerning the participating teacher and her classroom. The questions that were contained in this instrument are shown in figure 3.

Figure 3. Background Survey

I need to gather some information on your background. Please take a few minutes to fill out this survey.
Date _____

Personal Information:
Name _____
Address _____
Home Phone Number _____ School Phone Number _____

Educational Background:
Year graduated with BA or BS _____ Major _____
Institution granting your degree _____
Grades you are certified to teach _____
Subjects you are certified to teach _____
Advance Degrees _____ Field(s) _____
Dates and Institutions granting advanced degrees _____

School Information:
School _____
Is your school classified as an elementary school, middle school, or junior high school? _____
What grade levels does your school contain? _____

Teaching Information:
Number of years teaching _____
Present grade level that you teach _____
Number of years teaching at present grade level _____
Subjects that you currently teach _____

Additional Professional Experiences:
Are you familiar with the NCTM Standards?
If yes, please explain how you have been exposed to the NCTM Standards.

Have you participated in any projects/research studies in mathematics education?
If yes, please explain what the project/research study was and the role that you played.

Task Knowledge Inventory

Prior to the administration of each task to students, the participating teacher was asked to complete each task using as many different approaches as she could. The purpose of this was to determine whether the teacher had a broad enough knowledge of the mathematical concepts contained within these tasks to interpret diverse student responses.

Prediction Component

In order to determine whether the teacher acquired any new information with respect to her students' knowledge, the teacher was asked to predict how her students would perform on each task prior to administration. The teacher was asked to predict both the number of students that would acquire a correct answer and the number of students that would employ each of the approaches that she identified in the task knowledge inventory. The teacher also predicted the number of students that would not employ a previously identified approach. The discrepancies between the teacher's predictions and the students' responses was expected to suggest the availability of information concerning the students' knowledge that the teacher did not previously have.

Interview

In order to determine what information the teacher acquired and used with respect to student's knowledge, the teacher was interviewed as she examined students' responses to the area task. The first three questions in this interview permitted the teacher to select how she would discuss the responses and which responses she would discuss. This allowed the teacher to determine whether she would address responses at a group or individual level. The fourth question directed the teacher's attention upon selected student responses. Nine responses were selected randomly for each task. A month later, the same interview was completed concerning her students' responses to the average task. Two of the selected students were absent on the day that the average task was administered, resulting in seven student responses. The questions that comprise this interview are shown in figure 4.

Figure 4. Interview Questions

The first three questions were asked with respect to the larger group of response:

1. Were you surprised by your students performance on this task? (If yes, what surprised you?)
2. Were you surprised by the ways your students used to solve this task? (If yes, what surprised you?)
3. Did your students use any ways that you did not expect? (If yes, what were they?)

The question that follows was only asked with respect to the selected student responses.

4. Were you surprised by the selected students' performance on this task? (If yes, what surprised you?)

<Repeat question 4 for each selected student response>

5. Is there anything further that you would like to share about this experience?

Procedures

At the start of the study, the participating teacher was asked to complete the background survey and the task knowledge inventory. She then predicted the number of students that would answer the problem correctly, the number of students that would use each of the approaches she identified, and the number of students that would not employ one of the approaches that she identified.

The following day, the teacher administered the area task to her students. The directions that were read aloud to the students are shown in figure 5. The teacher was asked not to look at the student responses prior to the first interview. Students were provided 10 minutes to complete the task. The teacher was interviewed on the same day.

Within a month after the interview, the teacher was asked to complete the average task using as many different methods as she could. The administration of the task and the interview proceeded in the same manner as had been done for the area task. The set of students' responses that the teacher examined for the average tasks were selected to be from a different set of students than those used for the area task.

Figure 5. Directions to tasks

The italicized sentences are for your information and are not to be read aloud to students.

Read the following aloud to students:

Read the problem carefully before you begin working. These problems ask you to show your work or explain your answer. Your work or explanation should be clear enough so that another person could read it and understand your thinking.

You will be given 10 minutes to complete these problems. If you have any question raise your hand.

Please work quietly. You may begin now.

After 10 minutes:

Please stop working on these problems. Pass your papers forward.

Analysis

In order for a teacher to acquire information through the examination of students' responses, it is reasonable to assume that two conditions must hold. First, the teacher needs to have an adequate understanding of the task to support the information acquisition process. Without this type of understanding, the teacher would be unable to separate appropriate responses from inappropriate responses. Second, it is necessary to verify that the

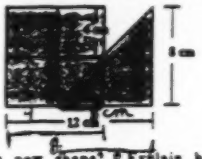
set of students' responses offers information that the teacher does not currently have. The first two sections that follow examine whether these conditions have been satisfied in the current study. Finally, the analysis of information acquisition and use is described.

Teacher's Understanding of Task

Prior to the students completing either task, the teacher solved the problems using as many approaches as she could develop. The teacher's approaches to the area task are shown in figure 6 and to the average task are shown in figure 7. All of the approaches that the teacher displayed are appropriate and resulted in correct answers. Another observation that can be made is that this teacher displayed a variety of different modes, referents, and relationships in her responses. This examination supports the assertion that the teacher in the proposed study has an adequate understanding of the concepts involved in these tasks to support the interpretation of her students' responses.

Figure 7. Teacher's Solutions to Area Task

Approach 1

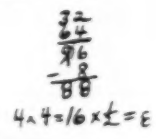


What is the area of the new shape? Explain how you found your answer. You may use the drawings in your explanation.

I determined the area of the part of the triangle that overlapped the larger square by cutting the square into fourths. The side measurements allowed me to do this (12 - 8 = 4 halfway pt. of length and the 4cm designated on the width). Each found is (64 ÷ 4) 16 cm². Half of that represents the overlapping triangle (8) and I subtracted this from the total area of both.

Area of new shape: 88 square centimeters.

Approach 2



actually draw lines to divide the square and rectangle into centimeter squares count the area of the overlapping triangle and subtract from total area

Approach 3

finding total area 8 × 12 = 96 and subtracting ½ of the total area 96 × ½ = 48 missing part = overlapping part

Approach 4

looking at the square / triangle relationship - breaking the triangle into smaller parts relative to the square to determine area of the overlapping triangle (completely)

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Figure 8. Teacher's Solutions to Average Task

Approach 1

A. What score must Anita get on Project 4 so that her average for the four projects is 17?

Answer: 19 You may draw your answer on the graph.

B. Explain how you found your answer.

15 , 18 , 16 average = 17

17 × 4 = 68 → To determine the total points to have an avg of 17.

15 + 18 + 16 = 49 → The total points thus far

68 − 49 = 19 → Subtracted needed points from points accrued.

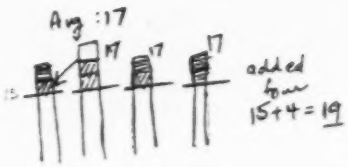
Approach 2

Guess & check

15 + 18 + 16 + 15 = 16.75 67÷4

15 + 18 + 16 + 19 = 68÷4 17

Approach 3



Approach 4

15 + 2 = 17

18 − 1 = 17

16 + 1 = 17

Need 2 extra points to get an average of 17

17 + 2 = 19

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Availability of Previously Unknown Information

The teacher had predicted that 60% of her students would correctly solve the area task. Only 36% of the students actually acquired a correct answer. Table 1 contains a summary of the discrepancy that existed between the teacher's predictions of the approaches that her students would employ and the approaches that her students did employ with respect to the area task. A student's approach was considered the same as a given teacher developed approach if the majority of relationships displayed in the response were also displayed in the teacher's approach.

Table 1

Teacher's predictions and student outcomes for the area task

	Percent Predicted (n=28)	Percent Employed (n=22)
Approach 1	25%	18%
Approach 2	18%	5%
Approach 3	0%	0%
Approach 4	25%	0%
Not previously identified correct approach	18%	41%
Incorrect Approach	14%	37%

Several observations can be made with respect to this table. First, 41% of the students' responses employed a correct approach of which the teacher had not previously identified. Another observation is that the teacher had predicted that one fourth of her students' would employ approach 4 and none of her students' employed this approach. Thirty-seven percent of her students employed an incorrect approach. The teacher had anticipated that only 14% would use an inappropriate approach. The above results support the conjecture that the students' responses contained information of which this teacher was not previously aware.

The teacher correctly predicted that 89% of her students would acquire a correct answer to the average problem. However, as illustrated in table 2, the teacher was not as accurate in predicting the approaches that her students would use. The teacher had anticipated that far more students would use approach 1 then actually did. Although the teacher anticipated that 11% of students would employ approach 3, none of the students used this approach. Many more students employed approach 2 than the teacher had predicted. Another observation is that the teacher was confident that all of her students would use a correct approach in the solution of this task. Eleven percent of students did not. Once again, the inconsistencies between the teacher's predictions and the outcomes suggest that the students' responses did offer information of which the teacher was previously unaware.

Table 2

Teacher's predictions and student outcomes for the average task

	Percent Predicted (n=28)	Percent Employed (n=28)
Approach 1	57%	17%
Approach 2	21%	61%
Approach 3	11%	0%
Approach 4	11%	11%
Incorrect Approach	0%	11%

Information Acquisition

Information acquisition was examined using two separate techniques. The first technique focused upon the examination of teacher "utterances". This technique will be described in the section that immediately follows. The second technique compares the information that was made available in the students' responses to the information that the teacher addressed during the interview. Each is described below.

Examination of Acquisition Utterances

An "utterance" will be used here to refer to the teacher's communication of a complete thought that may take the form of either a sentence(s) or a phrase(s). All of the teacher's utterances were coded as concerning information acquisition, information use, or other. Information acquisition refers to teacher statements concerning what they learned through the examination of a response. Information use refers to the teacher's statements that an action would take place based on the information acquired through the responses.

All information acquisition utterances were sub-coded. The teacher's information acquisition utterances were coded as referencing either a group of responses (Group) or a specific response (Individual). The utterances were then examined to determine whether the information was acquired through the examination of modes, referents or relationship. Additionally, it was observed that the teacher occasionally referred to the correctness of an acquired answer. Due to this observation, utterances that referenced the correctness of an answer were also tracked.

The first three questions in the interviews did not focus the teacher's attention upon any particular students' response. Rather, the teacher was permitted to select whatever aspects of the responses that she felt were important. The original intent was to examine the teacher's responses to the first three questions with respect to both tasks. However, due to a mechanical failure, it was only possible to examine the first three questions of the interview for the average task.

Modes, referents, relationships, and correctness of the answer were then crossed with groups and individuals resulting in table 3. The cells of table 3 reflect the proportion of respective teacher utterances in reaction to the first three questions that were devoted to modes, referents, relationships and the correctness of the obtained answer with respect to the average task. As can be observed in this table, the bulk of acquisition utterances were devoted to relationships and this attention focused heavily upon individual student responses.

Table 3

Teacher information acquisition utterance for the average task

(Total of 69 utterances)

	Group	Individual	Overall
Mode	.00	.04	.04
Referent	.00	.01	.01
Relationships	.35	.52	.87
Correctness of answer	.03	.03	.06
Other	.00	.01	.01
Overall	.38	.62	

Information Offered versus Information Discussed

As a precursor to this study, the researcher analyzed the set of selected students' responses to determine what information was made available with respect to modes, referents or relationships for each task. The researcher coded each set of selected student responses for the modes and referents that they contained. For both tasks, the responses were coded for three modes: symbolic (S), textual (T), and diagram (D). Symbols referred to responses that contained numbers and operators. Text referenced textual explanations and diagrams referred to responses that contained charts and illustrations. Responses were coded for as many modes as they contained. A similar analysis has been used in past research for the examination of modes (Magone et al., 1993; 1994).

Students' responses for each task were then coding according to the referent systems that they employed. In the area task, the students either used numerical solutions or segmented the diagram into parts. The student responses were coded as either numerical manipulation or physical decomposition. Most responses used a combination of referents and were coded as such. Two types of referents were also identified for the average task. Some students relied on numerical manipulation while others referenced the shifting of the bars on the chart. These referents were also coded as numerical or physical. Finally, each response was examined for the relationships that were reflected in students' responses. Some of the relationships that the students displayed were appropriate whereas others were inappropriate. Relationships were considered appropriate when they are

mathematically supportable. Relationships were classified as inappropriate when they are not mathematically supportable. Both appropriate and inappropriate relationships were noted. The results of this analysis are summarized in tables 4 and 5.

The audio tape of the entire interview was examined by the researcher to determine which aspects of each response the teacher successfully identified. A checklist was maintained for each task. The proportion of available modes that were successfully identified by the teacher for each common response is reported in these tables. These tables suggest the extent to which the teacher acquired the information that was available in the students' responses.

Table 4
Information Offered and Information Acquired for Area Task

I.D.	Modes	Referents	Relationships
01	Diagrams Text Symbols	Numbers Physical ✔	• Irregular figure is composite of regular shapes ✔ • Relationship between area of square and four small triangles • Removal of overlap
02	Diagrams Text	Numbers Physical	• Irregular figure is composite of regular shapes * Area of remaining segment of triangle
03	Diagrams ✔ Symbols	Numbers Physical	• Irregular figure is composite of regular shapes • Relationship between area of area of square and four small triangles • Removal of overlap
04	Symbols Text	Numbers	* Doubling of area of square • Area of 4x4 square is double that of right isosceles triangle with legs of length 2 • Removal of overlap
05	Diagrams ✔ Symbols Text	Numbers Physical ✔	• Composite of whole can be divided into individual units ✔ • Removal of overlap
06	Diagrams Text	Physical ✔	• Composite of whole can be divided into individual units ✔ • Recognized half units
08	Diagrams Text	Physical	Not interpretable ✔
09	Diagrams Text	Physical	• Irregular figure is composite of regular shapes * Use of perimeter in place of area
Pr tion	.11	.25	.22

Table 5
Information Offered and Information Acquired for Average Task

I.D.	Modes	Referents	Relationships
10	Diagrams Symbols Text ✔	Numbers ✔	• Recognition that average is bounded by range. • Appropriateness of average algorithm ✔
11	Diagrams Symbols Text	Numbers ✔	• Appropriateness of average algorithm ✔
12	Diagrams Symbols Text	Numbers ✔	* Average is half of something ✔
13	Text Symbols	Numbers	Unknown ✔
14	Symbols Text	Numbers ✔	• Appropriateness of average algorithm ✔
15	Diagrams Symbols Text	Numbers	• Relationship between average and the sum of the given scores ✔ * Unknown manipulation of numbers ✔
16	Diagrams Text	Physical	• Average requires leveling off
Proportion	**.06**	**.57**	**.78**

Information Use

Teachers' utterances were also examined to determine how the teacher perceived that she might use the information that she acquired. Since the teacher was examining the students' responses during the interview, it was not possible for her to have already applied the information to the classroom. The utterances that were previously identified as "use utterances" were sub-coded as relating to decision making or feedback.

For both interviews, the proportion of use utterances that were devoted to decision making and feedback crossed with group and individual were tabulated. These are shown in tables 6 and 7. The first three questions for the area task were not included in this examination due to a mechanical failure. The proportions for the area task were taken out of a total of 7 use utterances and the proportions for the average task were taken out of 14 use utterances. All feedback utterances were identified by the teacher as serving the specific purpose of assigning grades. An observation that can be made from these tables is that the majority of use utterances with respect to both tasks were devoted to decision making and impacted upon groups of students rather than individuals. This was especially true with respect to the Area task.

Table 6

Proportion of use utterances devoted to each aspect for the Area Task

Total: 7 use utterances

	Decision Making	Feedback	Overall
Group	.71	.14	.85
Individual	.14	.00	.14
Overall	.85	.14	

Table 7

Proportion of use utterances devoted to each aspect for the Average Task

Total: 14 use utterances

	Decision Making	Feedback	Overall
Group	.36	.21	.57
Individual	.29	.14	.43
Overall	.65	.35	--

Discussion

The analysis of the teacher's responses to the Task Knowledge Inventory suggest that the participating teacher did have an adequate understanding of the concepts that were being assessed in each task. This was reflected in her success in identifying four different methods that varied by mode, referents and relationships employed for completing each task. This further suggests that the participating teacher possessed an underlying knowledge of the different modes, referents and relationships that may make-up responses. Also, this teacher was aware, as reflected in her predictions, that students would use a variety of different approaches.

Another observation that can be made is that the students' responses did offer information to the teacher that she did not previously possess. This was evidenced by the differences between the teacher's predictions and the student outcomes. In both tasks, the teacher was unable to accurately predict the methods that her students would employ. In the area task, the teacher additionally inaccurately predicted that number of students that would produce a correct answer. If the teacher had a complete understanding of her students' knowledge with respect to the concepts reflected in this task, theoretically, she should be able to predict with a high degree of accuracy her students' outcomes. The discrepancies between the teacher's predictions and the student outcomes suggest that the responses contained information that the teacher did not already possess. A comment made by the teacher during the interview supports this assertion, "My students used much less sophisticated strategies than I expected. They understand the concept, but not at the level I desire".

The teacher's success in predicting the percent of students that would correctly respond to the average task can be interpreted in two manners. The teacher may have had a better understanding of her students' knowledge with respect to the concept of average than she did with respect to the area concept. Another interpretation is that the teacher's prior experience of inaccurately predicting her students' performance in the area task, resulted in less optimistic predictions for the average task. Regardless of which assertion is correct, the difference between the predicted methods with the methods employed supports the assertion that information was made available in the responses to both tasks that the teacher did not already possess.

In the information acquisition process, the majority of the teacher's utterances were devoted to the identification of relationships reflected in student responses. It was only possible to complete this analysis with respect to the Average task. The attention that this teacher devoted to the discussion of relationships was also evidenced in the comparisons that were made between the information offered in students' responses and the information that the teacher identified. In the Area task, far more of the available information was acquired with respect to relationships than was acquired analyzing responses to the Average task. This finding may either reflect the influence of using different tasks or the degree of the teacher's interest with respect to each concept assessed.

A difficulty that emerges in making-sense of the degree of attention that this teacher devoted to relationships is that it is unclear whether her attention was a natural response to the interpretation process or an artifact of the study. This teacher participated in two activities that proceeded and may have influenced her attention during the interpretation process. First, she constructed different solutions to the problems. In doing so, her attention was directly focused upon identifying the different relationships that could be established to acquire a correct answer. Second, in predicting how many of her students would use each approach, her curiosity may have been raised in the interpretation process to the examination of the relationships established. If either of these activities did direct the teacher's attention to relationships, it is difficult to conjecture as to what the teacher would have focused upon had these activities not been completed.

Another area which this teacher dedicated greater attention was to the referents used in the completion of the problems. In the area problem, the teacher commented on the students' use of physical referents. In the average problem, her attention was upon numerical solutions. This finding is most likely attributable to the nature of students' responses. In the average task, the majority of students used numerical referents. This left the teacher with little option in the type of referents that she could examine. Students' responses to the area task displayed greater diversity in the referents used. In this task, the teacher tended to mention referents when they were physical referents.

In both tasks, the students used a broad range of communication techniques. Yet, very little of the teacher's attention was devoted to the discussion of modes during the interview. There are several different ways in which to interpret this result. Since the given set of student responses displayed diverse modes, the development and use of diverse modes in the solving process may have been an earlier goal of this classroom. As the students demonstrated their proficiency in this area, the teacher may have redirected her attention away from this area of concentration. An alternative interpretation is that this teacher may have viewed modes as insignificant in informing her of students' knowledge. If this is the case, then the students use of different modes may be a reflection of the task structure rather than a reflection of true proficiency in this area. The results of this study do not conclusively support either conclusion.

Although the bulk of the teacher's attention during the acquisition process was upon the individual, her attention during the discussion of potential uses was upon the larger group. Either the teacher was only making use of the group level information that she acquired or she was aggregating individual information she acquired for group purposes. Another finding with respect to information use, was that this teacher tended to discuss how she would use the information to shape instructional decision making. A comment made during the interview appears to be inconsistent with this observation. The teacher stated that one of her short-comings in the assessment process was her tendency to use the information obtained through the assessment process for the sole purpose of grading. Yet, the majority of her use utterances were devoted to instructional decision making. This does not necessarily suggest that the teacher was inaccurate in reporting her past practices, but rather that the results presented here are a reflection of the teacher's efforts to apply the information to instructional purposes.

On a larger scale, this study suggests that the participating teacher was successful in acquiring useful information through the examination of her students' responses to the given open-ended tasks. The bulk of the information that was acquired at an individual level. Either information acquired on an individual level was aggregated to a group level or the teacher only considered the information that was acquired on group level for classroom purposes. The results further suggest that although this teacher did acquire information through the examination process, there was much more information offered then the teacher discussed. For example, the teacher provided little attention to the modes of communication that the students' employed.

Some cautionary notes should be made with respect to the interpretations of this study. This teacher had participated in the QUASAR rating sessions. This experience may have provided her with a better understanding of how to interpret open-ended responses then would be found in the general population of teachers. Additionally, she maintained an elementary mathematics specialist certificate. Teachers who do not share these

experiences may not display the same emphases in the interpretation and use processes. Future research is necessary to suggest the extent to which teachers with more typical experiences are successful in interpreting and using information acquired from students' responses to open-ended tasks.

References

Barr, R. (1988). Conditions influencing content taught in nine fourth-grade mathematics classrooms. *The elementary school journal*, 88 (4), 387-410.

Burton, L. (1984). Mathematical thinking: The struggle for meaning. *Journal for research on mathematics education*, 15 (1), 35-49.

Clark, C. M. & Peterson, P.L. (1986). Teachers' thought processes. In M. C. Wittrock (Ed.), *Handbook of Research on Teaching*, third edition (pp. 255-296). New York: Macmillan Publishing Company.

Hiebert, J. & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65-97). New York: Macmillan Publishing Company.

Kulm, G. (1994). *Mathematics assessments what works in the classroom.* San Francisco, CA: Jossey-Bass Inc.

Lane, S. & Silver, E. (1994, April). *The impact of reform-oriented instruction on students' mathematical proficiency: An examination of changes over time in middle school students' mathematical thinking and reasoning in the QUASAR project.* Paper presented at the annual meeting of the American Educational Research Association and the National Council on Measurement in Education, New Orleans.

Lesh, R. &. Lamon, S. J. (Eds) (1992a). *Assessment of authentic performance in school mathematics.* Washington, DC: American Association for the Advancement of Science.

Lesh, R. &. Lamon, S. J. (1992b). Trends, goals, and priorities in mathematics assessment. In R. Lesh and S. J. Lamon (Ed.), *Assessment of authentic performance in school mathematics* (pp. 3-15). Washington, DC: American Association for the Advancement of Science.

Magone, M. E., Cai, J., Silver, E. & Wang, N. (1994). Validating the cognitive complexity and content quality of a mathematical performance assessment. In R. J. Shavelson's (Guest Editor) Special Issue on Performance Assessment, *International Journal of educational research*, 21(3), 317- 340.

Magone, M. E., Wang, N., Cai, J. & Lane, S. (1993, April). *An analysis of the cognitive complexity of QUASAR's performance assessment tasks and their sensitivity to measuring changes in students' thinking.* Paper presented in the symposium "Assessing performance assessments: Do they withstand empirical scrutiny?" at the Annual meeting of the American Educational Research Association, Atlanta, GA.

National Council of Teachers of Mathematics. (1995). *Assessment Standards for School Mathematics.* Reston, Virginia: NCTM.

National Council of Teachers of Mathematics. (1991). Professional standards for teaching mathematics. Reston, Virginia: NCTM.

National Council of Teachers of Mathematics (1989). *Curriculum and Evaluation Standards for School Mathematics.* Reston, Virginia: NCTM.

National Research Council (1989). *Everybody counts* . Washington, D.C.: National Academy Press.

Pennsylvania Council of Teachers of Mathematics. (1995, March). *A positive attitude for performance assessment.* 44th Annual Meeting of the Pennsylvania Council of Teachers of Mathematics, Pittsburgh, Pennsylvania.

Streefland, L. (1992). Thinking strategies in mathematics instruction: How is testing possible? In R. Lesh & S. J. Lamon, *Assessment of authentic performance in school mathematics* (pp. 215- 245). Washington, DC: American Association for the Advancement of Science.

Resnick, L. B., Nesher, P., Leonard, F., Magone, M., Omanson, S., & Peled, I. (1989). Conceptual bases of arithmetic errors: The case of decimal fractions. *Journal for research in mathematics education*, 20 (1), 8-27.

Resnick, L. B. (1988). Treating mathematics as an ill-structured discipline. In R.I. Charles and E.A. Silver (Eds), *The teaching and assessing of mathematical problem solving* (pp. 32-60). Reston, Virginia: The National Council of Teachers of Mathematics Inc.

Shavelson, R. J. & Stern, P. (1981). Researching Teachers' Pedagogical thoughts, judgments, decisions, and behavior. *Review of educational research*, 51 (4), 455-498.

Stiggins, R.J. (1993). Two disciplines of educational assessment. *Measurement and evaluation in counseling and development*, 26 (1), 93-104.

Silver, E. A. (1993, March). *QUASAR Project Summary.* Pittsburgh, PA: Learning Research and Development Center, University of Pittsburgh.

Stiggins, R.J. (1991). Relevant classroom assessment training for teachers. *Educational measurement: Issues and practice, 10(1)*, 7-12.

Stiggins, R.J. (1990). Toward a relevant classroom assessment research agenda. *The alberta journal of educational research*, 36 (1), 92-97.

Thompson, A. G. & Briers, D. J. (1989). Assessing students' learning to inform teaching: the message in the NCTM's Evaluations Standards. *Arithmetic teacher*, 37 (4), 22-26.

Romberg, T. A., Zarinnia, E. A. & Collis, K. F. (1990). A new world view of assessment in mathematics. In G. Kulm (Ed.), *Assessing higher order thinking in mathematics* (pp. 21-38). Washington, DC: American Association for the Advancement of Science.

van den Heuvil-Panhuizen, M. (1995, April). *The tension between openness and certainty: An example of developmental research on assessment.* Paper presented as part of the symposium, "Theory and practice of authentic assessment in realistic mathematics education", at the annual meeting of the American Educational Research Association, San Francisco, California .

van Reeuwijk, M. (April, 1995). *Developing tasks to assess reasoning in early algebra.* Paper presented at the Annual Meeting of the American Educational Research Association, San Francisco.

Webb, N. L. (1992). Assessment of students' knowledge of mathematics: Steps toward a theory. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-370). New York: Macmillan.

Wiggins, G. (1989). A true test: Toward more authentic and equitable assessment. *Phi delta kappan*, 70 (9), 703-713.

Wiggins, G. (1988). Teaching to the (authentic) test. *Educational leadership*, 46 (7), 41-47.